IX. Maple: Computer Algebra System

Maple is a computer program for doing mathematics. It excels areas where computers can greatly benefit mathematics: algebraic manipulation, numerical computation, graphics, and programming.

To start up Maple use the command maple for the command line interface, or xmaple for the worksheet interface. Maple is also available on other platforms (like Windows95, Amiga, Mac) to the university community at a discount. Contact Andrzej Pindor at UTCC (address at end of document) for more details.

What you’ll see here is a brief introduction to Maple. Just enough so you can log on and explore for yourself. Maple’s help system is thorough and an important reference for every user. Just type ? at the prompt to start the help system.

> ?

To find help on any specific topic type ?topic and press return. This is the only Maple command that does not have to end in a semicolon (;). To get help on addition try

> ?addition

The official Maple manuals are The Maple Learning Guide, and the Maple Programming Guide. You can pick up a copy at the bookstore (or the department may have a copy you can look at). There are other excellent Maple books. Visit the Waterloo Maple Web Page (http://www.maplesoft.com) for a book list and other information about Maple.

At it’s simplest Maple is like a calculator. Type in any calculator operation (use brackets if necessary), end your command with a semi-colon and press return.

> 3* (97-282)+6/93;

\[-\frac{17203}{31}\]

Notice Maple (unlike your calculator understands exact rational arithmetic and automatically reduces answers to lowest form. Maple also understand constants like Pi, euler’s number E, and the imaginary unit I. If you want to force evaluation to a floating point use a decimal number, or use the evalf command. Maple is case sensitive, it’s important to type Pi not pi.
\[
0.5 \times \frac{22}{7};
\]
\[1.571428572\]

\[
\text{evalf}(\text{Pi});
\]
\[3.141592654\]

By default Maple uses 10 digits in it’s computation but you can request more for one specific computation through evalf or if you change the value of the global variable Digits to tell Maple how many digits to use normally.

\[
\text{evalf}(\text{Pi},20);
\]
\[3.1415926535897932385\]

\[
\text{Digits} := 25;
\]

\[\text{Digits} := 25\]

\[
\text{evalf}(E);
\]
\[2.718281828459045235360287\]

Notice we used := as the assignment operator. The = symbol is used for creating equations and does NOT do assignments. Maple can work with symbols (algebraic expressions) as well.

\[
p := x^3 - 27;
\]

\[p := x^3 - 27\]

Some useful functions for manipulating algebraic expressions (particularly polynomials) include factor, expand, combine, simplify, subs, simplify and normal.

\[
\text{factor}(p);
\]
\[(x - 3)(x^2 + 3x + 9)\]

\[> \text{subs(x=a-1,"});\]

\[\frac{(a - 4)((a - 1)^2 + 3a + 6)}{\text{expand("});}\]

\[a^3 - 3a^2 + 3a - 28\]

Notice " refers to the last output, "" the second last and """" the third last.

Maple has over 2000 functions and routines. For a list of basic functions try \texttt{?inifcns} (for initially defined functions). Essential functions include: \texttt{sqrt} (square roots), \texttt{exp} (exponential function), \texttt{log[b]} (logarithm base \(b\)), \texttt{abs} (absolute value), \texttt{argument} (argument of a complex number), and \texttt{mod} (modular arithmetic) and \texttt{sin}, \texttt{arccos}, \texttt{cosh} etc... (trig, inverse trig, hyperbolic and inverse hyperbolic functions).

You can put several commands on a single line. Just be sure to separate them by semicolons.

\[> \text{abs}(3+4*I); \text{argument}(3+4*I); 77 \text{ mod } 5;\]
\[> \text{cos}(\Pi/5); \text{log}[3](27);\]

\[5\]

\[\text{arctan}\left(\frac{4}{3}\right)\]

\[2\]
$\frac{1}{4} \sqrt{5} + \frac{1}{4}$

\frac{\ln(27)}{\ln(3)}$

> simplify(");

3

Of course Maple has standard calculus routines like diff (partial differentiation), int (integration), limit (two sided and one sided limits), series (generalized series), taylor (taylor series), and much more.

> diff(1/(x^6-1),x);

$-6 \frac{x^5}{(x^6-1)^2}$

> int("x");

$\frac{1}{6} \frac{1}{x - 1} - \frac{1}{6} \frac{1}{x + 1} + \frac{1}{6} \frac{-x - 2}{x^2 + x + 1} + \frac{1}{6} \frac{x - 2}{x^2 - x + 1}$

> normal(");

$\frac{1}{(x - 1) (x + 1) (x^2 + x + 1) (x^2 - x + 1)}$

> normal("\,\text{expanded}");

$\frac{1}{x^6 - 1}$

> limit("\,x=\text{infinity}");
\[
\int_0^\infty e^{-x^2} \, dx = \frac{1}{2} \sqrt{\pi}
\]

Notice the left hand side of the last command uses inert function \texttt{Int} (just displays doesn’t compute) as opposed to the active \texttt{int}.

Maple routines are often organized into packages of similar programs. There are packages for linear algebra (linalg), combinatorics (combinat), first year calculus (student), statistics (stats), group theory (group) and many many more. To load a package use the function with.

\begin{verbatim}
> with(combinat);

[Chi, bell, binomial, cartprod, character, choose, composition, conjpart, decodepart, encodepart, fibonacci, firstpart, graycode, inttovec, lastpart, multinomial, nextpart, numbcomb, numbcomp, numbpart, numbperm, partition, permute, powerset, prevpart, randcomb, randpart, randperm, stirling1, stirling2, subsets, vectoint]
\end{verbatim}

> permute(3,2);

\[
[[1, 2], [1, 3], [2, 1], [2, 3], [3, 1], [3, 2]]
\]

Maple has natural math style functions. For instance we use the arrow notation to define a function in Maple. We can invoke functions and differentiate them, too.

\begin{verbatim}
> f:= x -> x-sin(x^2);

f := x → x − \sin(x^2)
\end{verbatim}

> f(4); f( sqrt(Pi));
\[ 4 - \sin(16) \]

\[ \sqrt{\pi} \]

\[ > \text{fprime} := \text{D(f)}; \]

\textit{fprime} := x \rightarrow 1 - 2 \cos(x^2) x

Maple has extensive graphical capabilities including 2D and 3D plotting and animation. The plots package has many more routines including other coordinate systems. The following command will draw the function \( \sin \) between -\( \pi \) and \( \pi \).
Maple has a full programming language. Some people compare it to Pascal but really it draws features from many languages. Maple has special constructs to take advantage of the structure of mathematical objects. For instance of course there is a for loop construct. In Maple an optional way to use this is the for in loop.

```maple
> mylist:=[71,73,93,103];
> for num in mylist do
> isprime(num);
> od;
```

```
mylist := [71, 73, 93, 103]
```
Procedures are easy to create in Maple. In the following example we make a procedure to compute fibonacci numbers and then we initialize it.

```maple
fib := proc(n) option remember;
if type(n, posint) then
  fib(n-1)+fib(n-2)
else
  ERROR('input not a positive integer')
fi;
end;
fib(0) := 0; fib(1) := 1;
```

```
fib := proc(n)
  option remember;
  if type(n, posint) then
    fib(n-1) + fib(n-2)
  else
    ERROR('input not a positive integer')
  fi
end

fib(0) := 0

fib(1) := 1
```
Notice the option remember. This tells Maple to remember the results of any computations done by fib. This makes computing fibonacci numbers much more efficient.

\[ > \text{fib}(17); \]

\[ 1597 \]

\[ > \text{fib}(\pi); \]

\[ \text{Error, (in fib) input not a positive integer} \]

UTCC offers free introductory Maple courses to the U of T community. Contact Andrzej Pindor (email: andrzej.pindor@utoronto.ca, phone: 978-5045) for details.